

The Role of trapped neutrino in dense stellar matter and kaon condensation

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Abstract:

We investigate the effect of neutrino trapping on the kaon condensation process and Equation of State (EOS) in a newly formed neutron star matter which is less than several seconds old. Using nonlinear relativistic mean field model, we find that the presence of neutrino shifts the threshold density for kaon condensation and muon production to much higher density. We also studied the pressure and energy density of the system and found that the presence of neutrino stiffens the EOS which may be responsible for the delayed explosion mechanism for supernovae.

I. Introduction:

The presence of neutrino is an essential part of gravitational collapse, supernovae and protoneutron stars. The scenario of type II supernovae is strongly marked by the dynamical behaviour of stellar medium under neutrino trapping effects. During the supernovae collapse phase, a large number of neutrinos are being produced by the electron capture process. So, just after few seconds of a

new born protoneutron star, the neutrino mean free path becomes larger than collapsing time scale marked by $\tau_h = (G\rho)^{1/2}$ [G is the gravitational const, $\rho \rightarrow$ density of the core] . Beyond densities of about $10^{13} \text{ g.cm}^{-3}$, neutrinos being trapped within the matter are unable to propagate on dynamical scale. These trapped neutrinos become an important ingredient in discussing the stellar matter Equation of State (EOS). So, in this case the beta equilibrium condition is being altered i.e

$$n + \nu_e \rightleftharpoons p + e$$

is established in a characteristic of time scale $\tau_\beta = 1/\sigma_0 c \rho_\nu = 10^{-15} - 10^{-18} \text{ s}$, which is a function of trapped neutrino density ρ_ν ($\sigma_0 \rightarrow$ typical cross section for ν -n process). Thus the composition of matter is affected and the threshold density for kaon condensation gets affected.

The inner core continues to collapse and higher values of nuclear densities are reached. A shock front is generated at the reversion of inner core collapse. So, characteristic equation of state determines the shock front. In the presence of neutrinos, the threshold density for occuring kaon condensation is shifted to higher density. So, the role of neutrino trapping is claimed to explain the delayed explosion mechanism. Properties of trapped neutrinos are already been discussed in many works [1,2].

Here we discuss the kaon condensation and other properties of neutron star matter under the trapped neutrino condition.

The idea that above some critical density, the ground state of baryonic matter may contain a Bose-Einstein condensate of negatively charged kaons was given first by Kaplan and Nelson [3]. Using an $SU(3)_C \times SU(3)_F$ chiral lagrangian , they showed that around $\rho \simeq 3\rho_0$ [$\rho_0 \rightarrow$ equilibrium nuclear matter density], kaon condensation is energetically favourable. Physically, the attraction between K^- mesons and nucleons increases the density and lowers the energy of the zero momentum state . A condensate forms when this energy becomes equal to the kaon chemical potential μ .

The presence of kaon condensation softens the equation of state which lowers the maximum mass of neutron star. In neutrino free dense matter, the density at which this condensation takes place is typically $\simeq 4\rho_0$ (ρ_0 denotes the

equilibrium density) which is much less than the density of the core of neutron star. So, K^- condensate is expected to be present in the core of the star.

Brown and Kubodera group [4] attempted the kaon condensation through s-wave interaction of kaons with nucleons and found rapid rise of condensate amplitude with density. Later few attempts [5,6] have been made to investigate the possibility of kaon condensation in the core of neutron star through p-wave interaction of kaons with nucleons and found the small values of kaon amplitude. Recently, Prakash group [7] improved this analysis of kaon condensation of neutron star matter including hyperons from relativistic mean field model. They also observed the rapid rise of condensate amplitude beyond threshold and the effect of hyperons on the threshold which shifts the threshold for condensation towards higher density. Very recently, Chiapparini et al [8] studied the EOS of neutron star matter under trapped neutrino without kaon condensation and hyperons. They studied the matter with different electron-lepton fraction value and found almost no effect on Equation of State.

Here in this work, we extend our analysis of neutron star matter (n-p-e- μ) including kaon condensation under neutrino trapping condition. We study the effect of neutrino trapping on the threshold density for kaon condensation in the core of dense stellar matter.

We study the matter from relativistic mean field model where the hadronic and leptonic densities are calculated in a self consistent way in β equilibrium under two conditions. One is, charge neutrality and the second one being the total baryon number conservation.

II. The lagrangian

The hadronic fields are considered here in relativistic mean field model in which baryons interact via the exchange of σ -, ω - and ρ - mesons with nonlinear self energy terms.

The total lagrangian is,

$$\mathcal{L}_{\text{total}} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_L$$

where \mathcal{L}_B , \mathcal{L}_M and \mathcal{L}_L describe the baryonic, mesonic and leptonic sector respectively and given by,

$$\mathcal{L}_B = \sum_B \overline{\psi}_B (i\gamma^\mu \partial_\mu - g_{\omega B} \gamma^\mu \omega_\mu - g_{\rho B} \gamma^\mu b_\mu \cdot \tau - m_B + g_{\sigma B} \sigma) \psi_B$$

$m_B \rightarrow$ vacuum baryon mass.

In the mesonic sector,

$$\begin{aligned} \mathcal{L}_M &= \frac{1}{2}(\partial_\mu \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{3}bM(g_\sigma \sigma^3) - \frac{1}{4}c(g_\sigma \sigma^4) \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{1}{2}m_\rho^2 \rho_\mu \rho^\mu + \mathcal{L}_K \end{aligned}$$

$$\mathcal{L}_L = \sum_{l=e,\mu,\nu} \overline{\psi}_l$$

We treat the meson in the mean field approximation and kaplan-Nelson SU(3)XSU(3) chiral lagrangian and kaon-baryon interaction for kaon.

$$\begin{aligned} \mathcal{L}_K &= \frac{1}{4}f^2 Tr \delta_\mu \delta^\mu c Tr m_q (u + u^\dagger - 2) + i Tr \overline{\psi}_B \gamma^\mu [V_\mu, \psi_B] \\ &+ a_1 Tr \overline{B} (\xi m_q \xi + h.c) B + a_2 tr \overline{B} B (\xi m_q \xi + h.c) + a_3 \{Tr(\overline{B} B)\} \end{aligned}$$

U is the nonlinear field involving pseudoscalar meson octet which retains only K^+ contributions.

$f \rightarrow$ is the pion decay const and C, a_1, a_2, a_3 are constants.

We have taken $a_1 m_s = -67.0$ MeV; $a_2 m_s = 134.0$ MeV and $a_3 m_s = -222.0$ MeV.

The effective mass m_B^* in the presence of kaon condensate is given by,

$$m_B^* = m_B + g_{\sigma B} \sigma + [(a_1 + a_2)(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) + 4a_3] m_s \sin^2 \frac{1}{2} \theta$$

and the chemical potential μ_B is given by,

$$\mu_B = \nu_B + g_{\omega B}\omega_0 + g_{\rho B}\tau_{3B}b_0 - (Y_B + q_B)\mu \sin^2 \frac{1}{2}\theta$$

where τ_{3B} is the $\frac{1}{2}$ -component of isospin of the baryon (here only p and n) and ν_B is given in terms of fermi momentum k_{FB} through

$$\nu_B = \sqrt{k_{FB}^2 + m_B^{*2}}$$

Here m_s is the scalar meson mass and θ is the kaon condensate amplitude. Thus the particle density ρ_B are obtained as,

$$\rho_B = \frac{\gamma}{6\pi^2} k_F^3$$

The degeneracy factor $\gamma = 2$ for massive particle and $=1$ for neutrino. and for kaon the density is,

$$\rho_K = f^2(\mu \sin^2 \theta + 4e \sin^2 \frac{1}{2}\theta)$$

with

$$e = \sum_B (Y_B + q_B) \rho_B / (4f^2)$$

For meson,

$$\mu_l = \sqrt{k_l^2 + m_l^2}$$

$l = e, \mu$

and $\mu_{\nu_e, \bar{\nu}_\mu} = k_{\nu_e, \bar{\nu}_\mu}$

Charge neutrality for the matter is conserved ,

$$\rho_p = \rho_e + \rho_\mu + \rho_K$$

we explore the effect of neutrino trapping by fixing electron-lepton fraction ,

$$Y_{le} = \frac{\rho_e + \rho_{\nu_e}}{\rho_B} = Y_e + Y_{\nu_e}$$

at total baryon density.

parindent 2pt So, we solve the \mathcal{E} uler- \mathcal{L} agrange equation for the baryon and meson field where the scalar density is obtained from,

$$\rho_B^s = \frac{\gamma}{2\pi^2} \int \frac{m_B^*}{\sqrt{k_B^2 + m_B^2}} k^2 dk$$

the condensate amplitude θ is found from ,

$$\mu^2 \cos\theta + 2e\mu - m_k^2 - d_1 = 0$$

where

$$2f^2 d_1 = \sum_{B=n,p} [(a_1 + a_2(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) + 4a_3] m_s \rho_B^s$$

Here we have used the coupling parameter values taken from Ref.[9].

For leptons, the electron and muon are governed by the chemical potential $\mu_e = \mu_\mu = \mu$.

The threshold density ρ_c for condensation is determined by setting $\theta = 0$ in the above equation, i.e

$$\mu^2 + 2e\mu - m_k^2 - [2a_1 \rho_p^s + (2a_1 + 4a_3)(\rho_p^s + \rho_n^s)] \frac{m_s}{2f^2} = 0$$

Once the effective mass for nucleon is obtained from self consistent solution, the total pressure density can be calculated as,

$$\begin{aligned} p &= \frac{\gamma}{6\pi^2} \sum_B \int_0^k dk k^2 \frac{k^2}{\sqrt{k^2 + m_B^{*2}}} - \frac{1}{2} m_\sigma^2 \sigma^2 \\ &+ \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 - \frac{1}{3} b \sigma^3 - \frac{1}{4} c \sigma^4 \\ &+ \frac{1}{3\pi^2} \sum_{\text{leptons}} \int_0^k dk k^2 \frac{k^2}{\sqrt{k^2 + m_L^2}} \end{aligned}$$

The contribution to pressure from neutrino is given by,

$$p_\nu = \frac{1}{24\pi^2} \mu_{\nu e}^4$$

Results and Discussions

We have studied the possibility of kaon condensation in nucleon only matter (without hyperon) under the trapped neutrino condition. **Figure 1a** shows the relative particle fraction ρ/ρ_B in the case when neutrino is absent. It shows the proton-electron degenerated curve to be split into two because of muon production. When the electron fermi momentum reaches the muon mass, muons are being produced. In the absence of neutrino, the threshold density for muon production occurs just near the normal nuclear density $\simeq 1.05\rho_0$. But the presence of neutrino shifts this threshold density to much higher density. **Figure 1b** shows that this threshold is shifted to $\rho \simeq 5\rho_0$ when neutrinos are being trapped in the matter for a typical value of final collapse $Y_{le} = 0.3$.

The electron neutrinos being produced in this system are pauli blocked, so more neutrons will decay in order to preserve Y_{le} at constant value. As a result, the matter becomes a protoneutron star. The presence of neutrino shifts the threshold density for kaon condensation to much higher density $\rho \simeq 8\rho_0$ for $Y_{le} = 0.3$ where normal threshold density for possible kaon condensation is $4\rho_0$ without neutrino trapping.

Here the kaon condensation process depends on two factors, the first being the behaviour of the scalar field $g_\sigma\sigma$ in the matter and the second being the strangeness content of the nucleon related to the magnitude of a_3m_s . The larger the magnitude of a_3m_s , the lower is the value of threshold density ρ_c .

It is also seen that though neutrino plays a major role in determining threshold density for kaon condensation but the condensate amplitude grows rapidly and at that stage the leptons play only a minor role only to preserve the Y_{le} constant. Since the threshold density for possible kaon condensate is very high

, so this may not be present in the core of protoneutron star system. We have plotted the nucleon effective mass, scalar field and electron chemical potential against density in **Figs. IIa & IIb** showing the behaviour in dense matter. The scalar field is less sensitive to density and almost constant in trapped neutrino matter which effectively decreases the kaon chemical potential. For typical final collapse value of $Y_{le} = 0.3$, $\mu \simeq 49$ MeV in contrast to $\mu \simeq 108$ MeV untrapped value.

The electron capture processes which proceed with density becomes stopped when trapped neutrinos settle into the matter. The immediate consequence is the decrease in μ values which significantly delay the kaon condensation until high density. This feature is significant for the evolution of neutron star at the early stage. Though kaon condensation softens the equation of state, but in trapped case the delayed kaon condensation gives rise to more pressure. As a result, the equation of state becomes more stiff in this case. **Fig. III** compares the pressure with and without kaon condensation in trapped neutrino matter with that pressure in case of free neutrino matter. As long as neutrinos remain trapped in the matter, the overall pressure is very high and gets softened when neutrinos diffuse out from the system. The Y_{le} drops out and threshold density for kaon condensation decreases. So kaon could appear in the matter which in turn softens the EOS of the matter. This may be the reason for delayed explosion mechanism of supernovae.

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Figure Captions:

1. **Fig. Ia.** The relative particle population of nucleons as function of total baryon density [ρ_B/ρ_0 , ρ_0 is equilibrium nuclear density] in neutrino free dense matter (without hyperon)
2. **Fig. Ib.** The relative particle population of nucleons as function of total baryon density for the value of electron-lepton fraction Y_{le} under trapped neutrino condition.
3. **Fig. IIa.** The nucleon effective masses, the kaon chemical potential $\mu = \mu_e$ and the scalar field $g_\sigma\sigma$ in free neutrino matter.
4. **Fig. IIb.** The nucleon effective mass, the kaon chemical potential $\mu = \mu_e - \mu_{\nu_e}$ and the scalar field for $Y_{le} = 0.3$
5. **Fig. III.** Pressure with and without kaon condensation in free neutrino case compared with the pressure with kaon condensation under trapped neutrino.

Table I. Coupling Constants used in this calculation

$(g_\sigma/m_\sigma)^2$	$(g_\omega/m_\omega)^2$	$(g_\rho/m_\rho)^2$	b	c
10.138	13.285	4.975	0.003478	0.01328

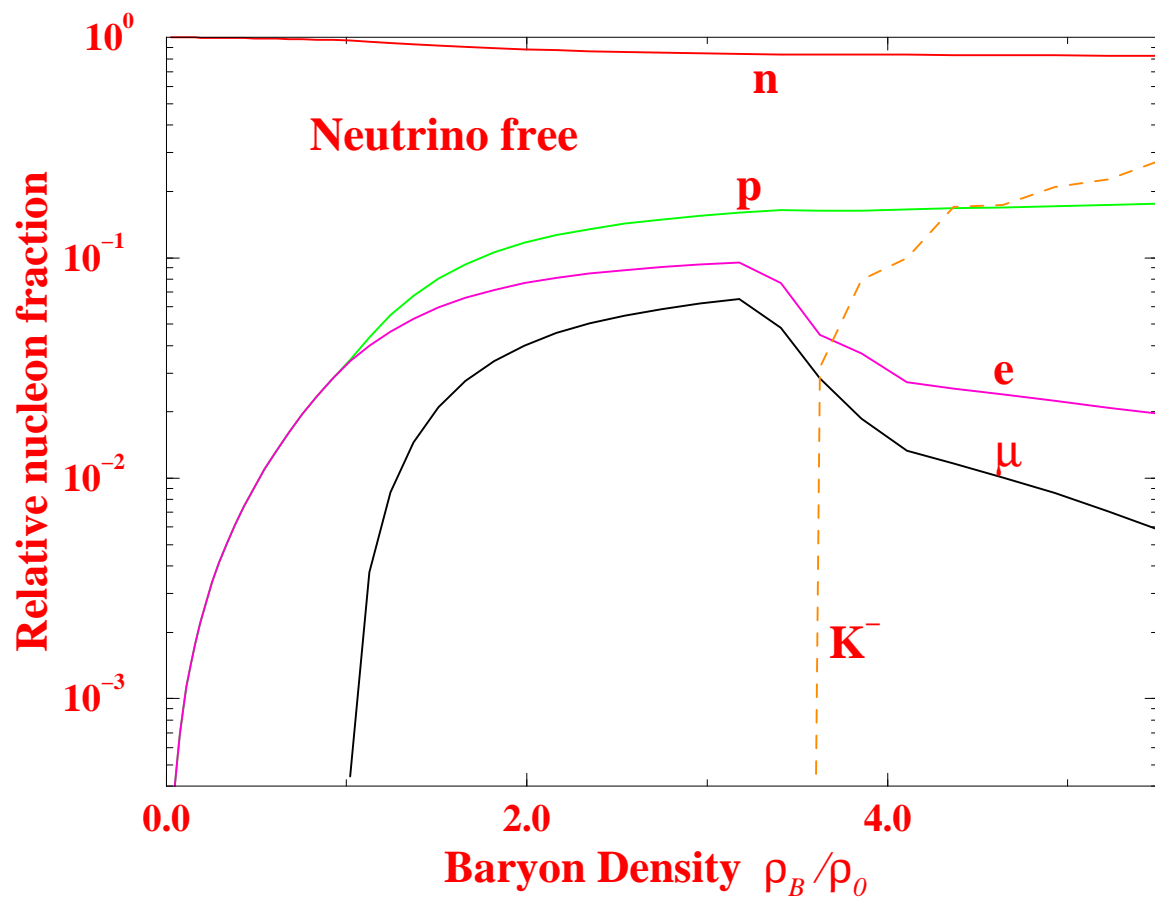


Fig. 1a

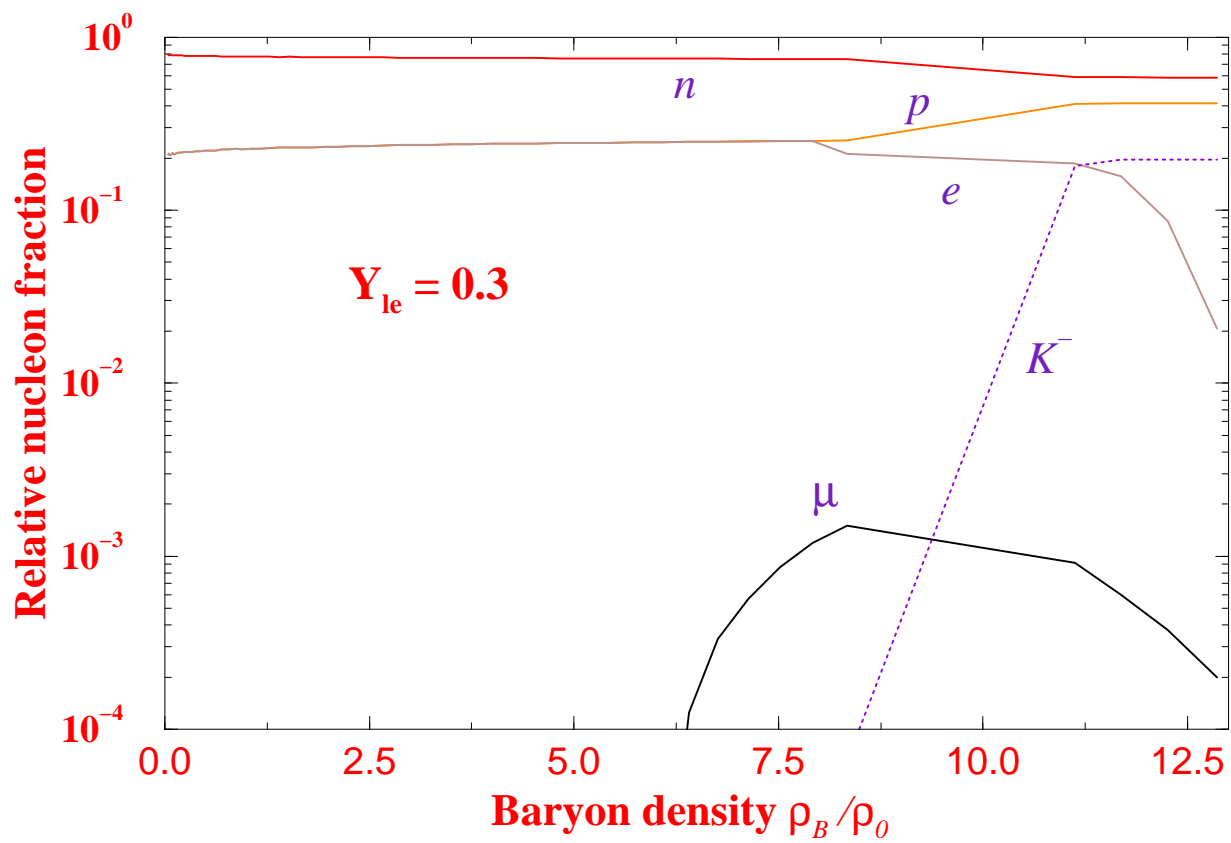


Fig. Ib

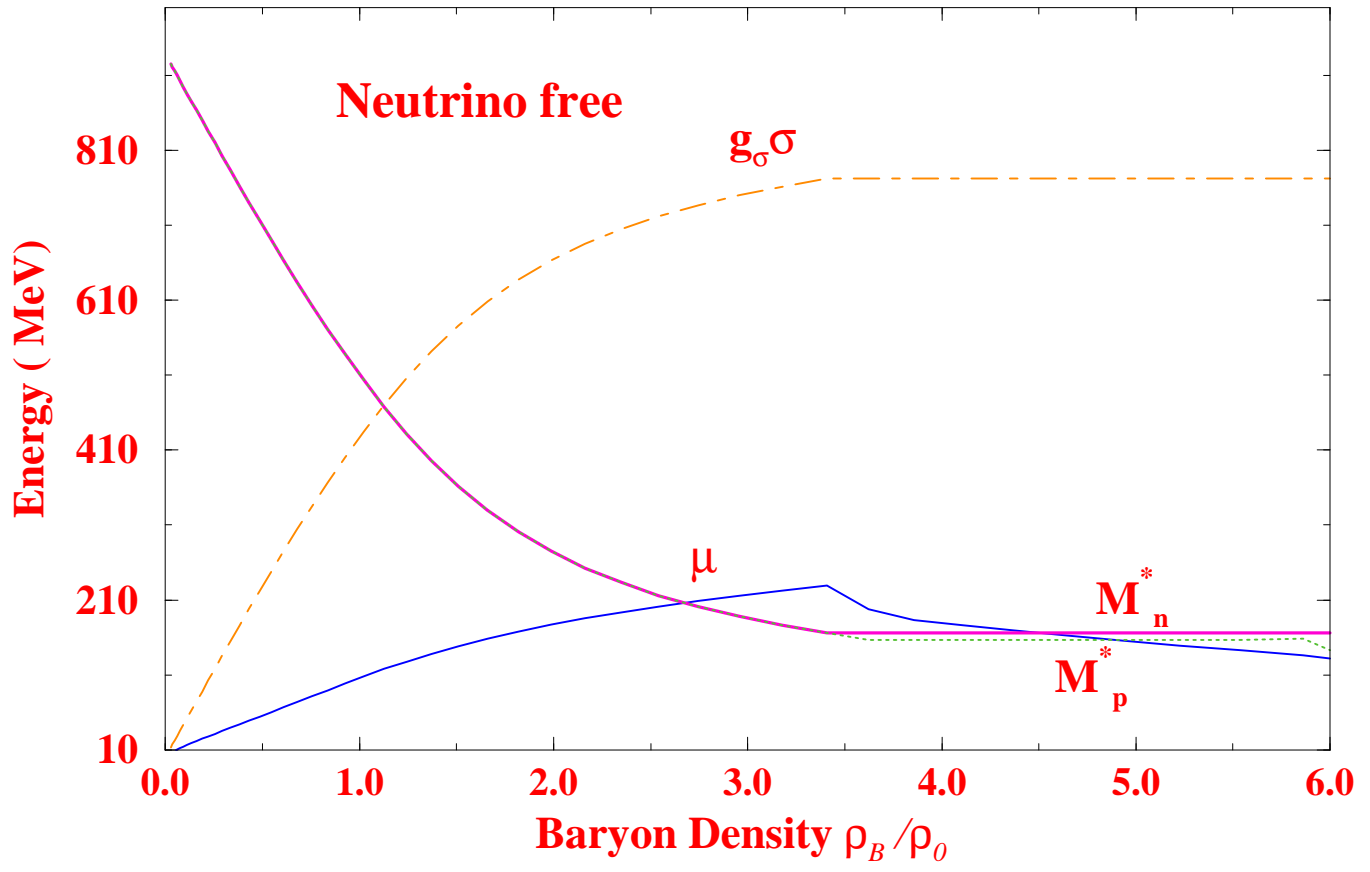


Fig. IIa

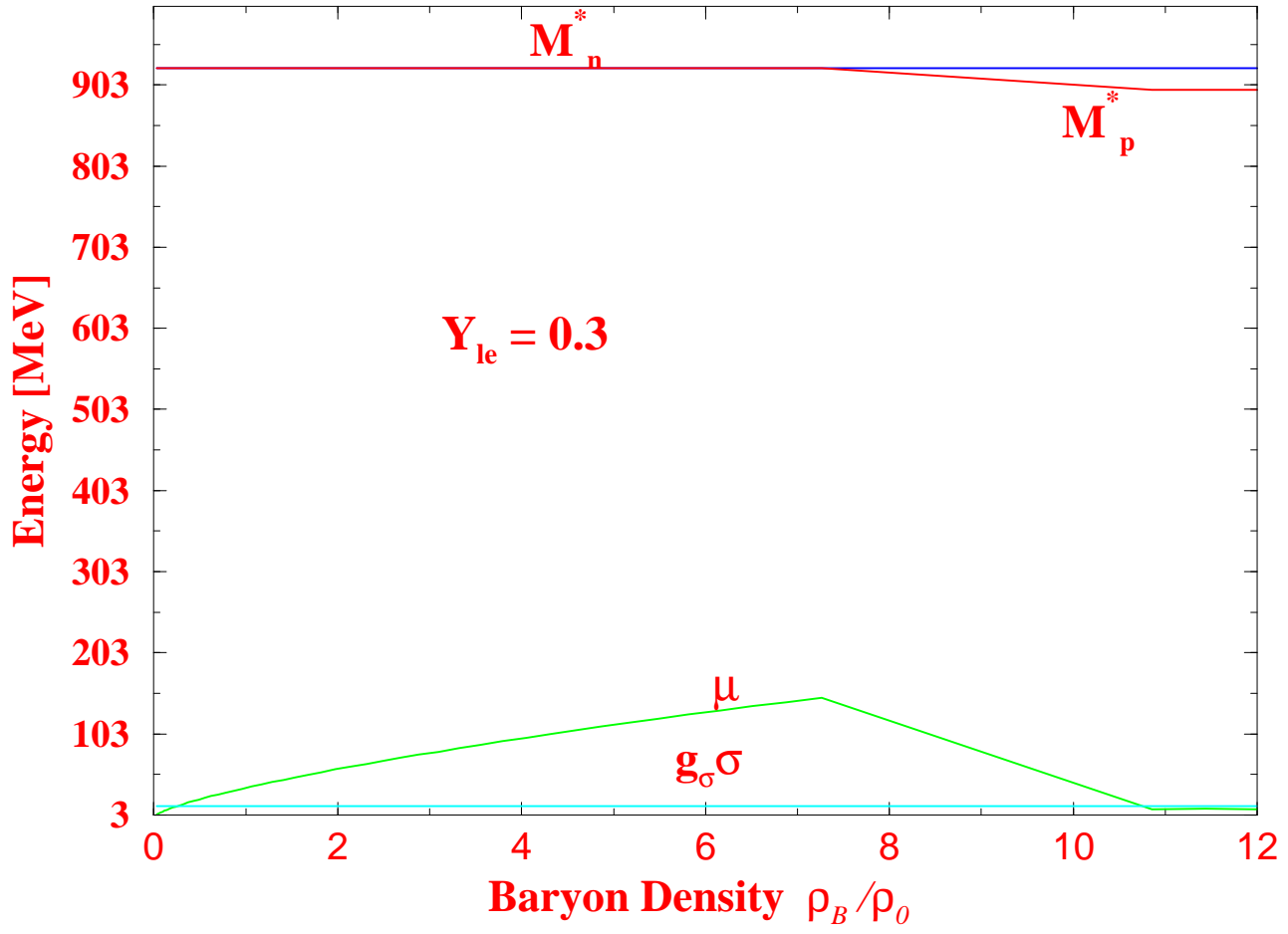


Fig. IIb

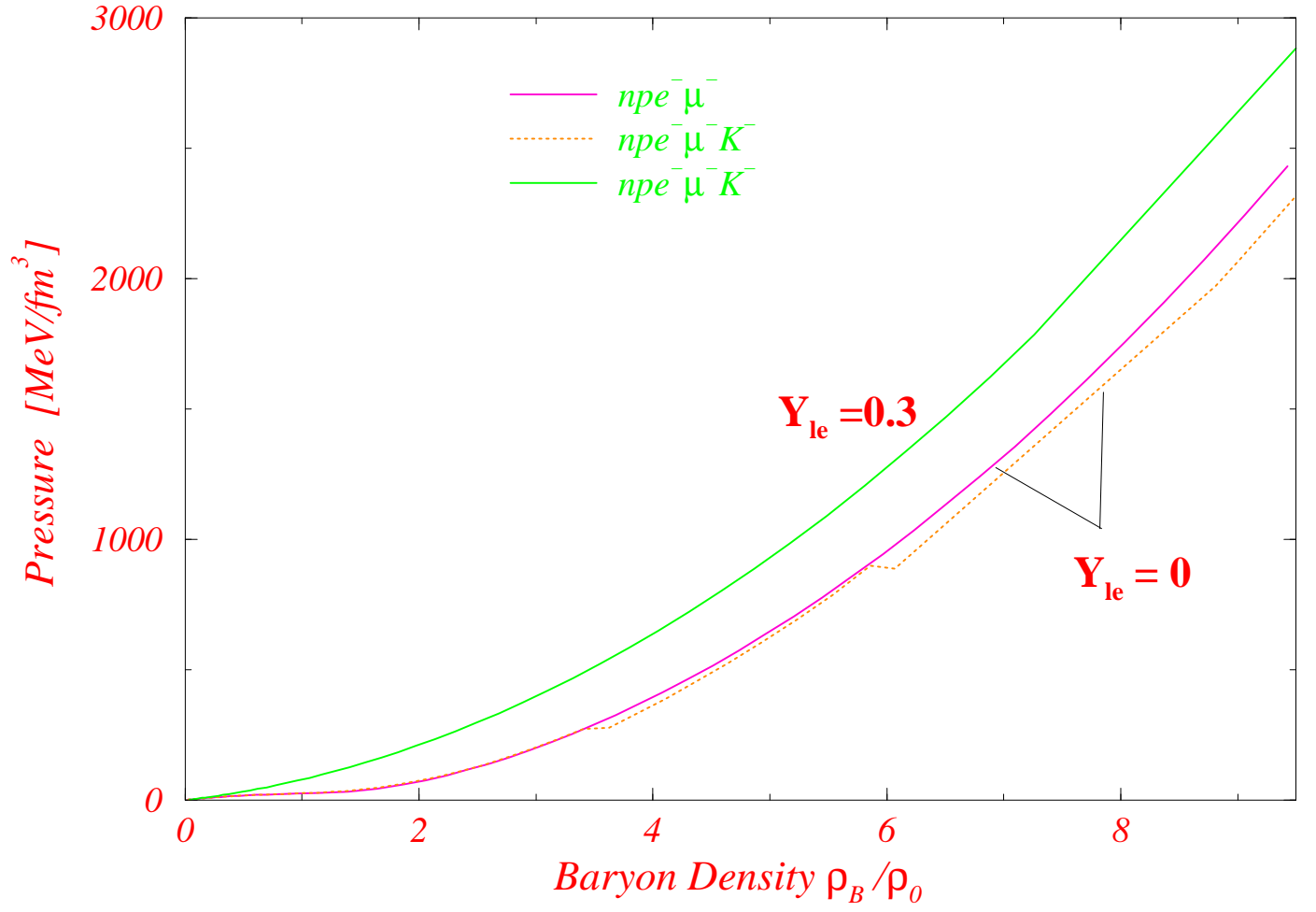


Fig. III